Conclusions

Expressions for the analytical temperature distribution and efficiency of a heat pipe fin were derived and compared with experimental data. Two heat pipe fin cases were studied, flush mounted and inserted into the object. The derived expressions are useful as a design tool. They allow the designer to compare heat pipe fins to standard fins before doing a detailed heat pipe design.

Further research could focus on variations in heat pipe design and geometry. These could include nonconstant properties and boundary conditions. For this work, the wall cross-sectional area, the internal convection coefficient, and the external convection coefficient were assumed constant. The impact of these assumptions on accuracy should be investigated. It would also be interesting to extend the analysis to fins in radiation environments.

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Forced Convection Heat Transfer to Phase Change Material Slurries in Circular Ducts

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Introduction

PHASE change material (PCM) slurries are being considered for use in temperature control and thermal energy storage systems. However, theoretical models for heat transfer in such slurries have received relatively little attention. Chen and Chen[1] used a Dirac δ-function-based analytical model and a perturbation method to investigate the augmentation of heat transfer for a steady, laminar PCM slurry flow above a flat plate with constant wall temperature. Charunyakorn et al.[2] developed a numerical model for heat transfer to PCM slurries for flows between parallel plates and in circular ducts for various boundary conditions. Goel et al.[3] conducted an experimental study to verify the model of Charunyakorn et al.[2] for the case of flow in a circular tube with a constant heat flux boundary condition. They found that the experimental results agreed only qualitatively with the numerical prediction, with the difference between the two being of the order of 45%. Zhang and Faghri[4] modified the model of Charunyakorn et al.[2] to include the effects of the crust of the microcapsules, initial subcooling, and the width of the phase change temperature range. They found that the differences between their numerical results and the experimental results of Goel et al.[3] could be reduced or eliminated entirely by incorporating these three effects in the Charunyakorn et al.[2] model.

Previous theoretical models use a complicated source term or special analytical techniques so that they cannot be readily used in commercial computational fluid dynamics (CFD) packages. Also, when the phase change process occurs over a finite temperature range (for example, with multicomponent substances), the effective specific heat of the material may vary as a function of temperature. The effects of this variation on the heat transfer process have not been investigated to date. To overcome these limitations, an effective specific heat capacity model for heat transfer to a PCM slurry is presented. This model does not include a source term and is more easily implemented in standard CFD packages. Instead, various forms of the specific heat functions have been considered to account for the phase change effects, and numerical results have been obtained using the model.

Model Formulation

It is advantageous to consider a standard problem at this stage because the primary goal is to develop and to evaluate a new model for heat transfer with PCM slurries. Once the model verification is complete, it will be possible to use it to study more complex heat transfer processes as required. Thus, the problem of laminar heat transfer to a PCM slurry flowing in a circular duct with constant wall temperature T_w is considered in this Note. The flow is assumed to be fully developed, and the slurry enters the heated section at a temperature T_i that is equal to or below the melting point of the PCM. The volumetric concentration of the suspended PCM is less than 20–25% so that the flow is essentially Newtonian. The density of the PCM in both the liquid and solid phases is approximately equal to that of the suspending fluid so that the particles can be assumed to be neutrally buoyant and the slurry density can be treated as constant. The sizes of the PCM particles are much smaller than the radius r_0 of the tube so that the suspension behaves as a homogeneous fluid and the effects of the particle free layer next to the wall are negligible. The flow rate (with mean velocity U_m) is assumed to be sufficiently high so that there is no separation of the slurry constituents but is low enough so that viscous dissipation and axial conduction effects are negligible. Note that the basic assumptions here are similar to those used in previous studies.[2,4]

Based on the preceding description of the model, a simplified form of the energy equation is used to model the heat transfer process. The phase change effects are included in the energy equation through the specific heat capacity of the slurry, which is taken as a function of temperature. The thermal conductivity k_e is also taken as a function of the radial position to account for microconvective effects induced by the PCM particles suspended in the fluid. The energy equation can, therefore, be written as

$$2 \cdot Cp \cdot \rho \cdot U_m \left[1 - \left(\frac{r}{r_0}\right)^2\right] \frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \cdot k_e \frac{\partial T}{\partial r}\right)$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r \cdot f \cdot k_b \frac{\partial T}{\partial r}\right) \tag{1}$$

with the following initial and boundary conditions:

$$T = T_i \quad \text{at} \quad x = 0, \qquad \frac{\partial T}{\partial r} = 0 \quad \text{at} \quad r = 0$$

$$T = T_w \quad \text{at} \quad r = r_0 \tag{2}$$

where k_b is the static bulk thermal conductivity and f is the microconvection-related enhancement factor that has been represented by piecewise linear functions[5] based on the general relation developed by Charunyakorn et al.[2]

The specific heat function must be derived carefully because it is the main part of this new model. As a first step, it is assumed that the specific heat capacity of the PCM, Cp_s, is the same in both the solid and liquid phases. Thus, in the absence of phase change, the specific heat of the slurry, Cp_{wo}, is calculated by using the mass fraction Cm of the PCM:

$$Cp = Cp_{wo} = Cm \cdot Cp_s + (1 - Cm) \cdot Cp_f \tag{3}$$

During the phase change process, the effective specific heat of the PCM will be a function of temperature (Fig. 1). In this temperature range, the effective specific heat $Cp = Cp_{\mathrm{sl}}$ can be related to the latent heat h_{fs} by the following equation:

$$h_{fs} = \int_{T_1}^{T_2} Cp_{\mathrm{sl}} \cdot \mathrm{d}T \tag{4}$$

Four different specific heat functions have been considered in the present work. To represent symmetric distributions, rectangles and sinusoidal curves were chosen, whereas two oppositely oriented

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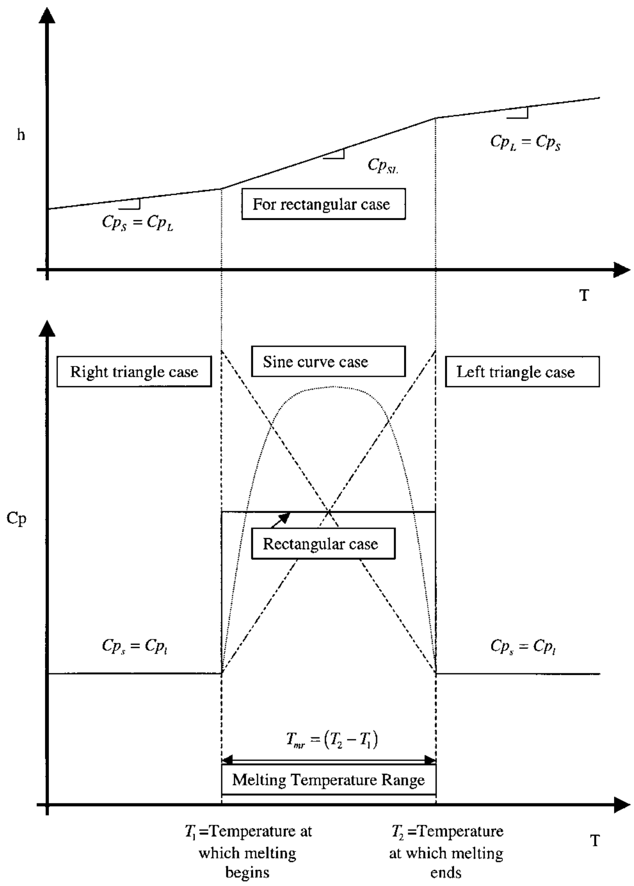

Fig. 1 Variation of effective specific heat with temperature: four different distributions of the specific heat with equal area and constant width.

right triangles were used for the asymmetric distributions (Fig. 1). These functions are idealized representations of the specific heat functions of real substances and are chosen because they are relatively easy to implement in a numerical model. The effective specific heat for each of these cases can then be written as follows.

Rectangle:

$$Cp_{sl} = h_{fs}/(T_{mr}) \tag{5}$$

Right triangle:

$$Cp_{sl} = Cp_s + \left[2 \cdot (h_{fs}/T_{mr}^2 - Cp_s/T_{mr})\right] \cdot (T - T_1) \tag{6}$$

Left triangle:

$$Cp_{sl} = Cp_s + \left[2 \cdot (-h_{fs}/T_{mr}^2 + Cp_s/T_{mr})\right] \cdot (T - T_2) \tag{7}$$

Sine curve:

$$Cp_{sl(T)} = Cp_s + \{\pi/2 \cdot (h_{fs}/T_{mr} - Cp_s) \cdot \sin \pi[(T - T_1)/T_{mr}]\} \tag{8}$$

The specific heat of the slurry undergoing phase change, Cp_{wp}, is then calculated by using the following equation:

$$Cp_{wp} = Cm \cdot Cp_{sl(T)} + (1 - Cm) \cdot Cp_f \tag{9}$$

The problem has been nondimensionalized prior to obtaining the solution using the following nondimensional variables:

$$r_1 = \frac{r}{R}, \qquad u_1 = \frac{u}{U_m}$$

$$x_1 = \frac{x}{R \cdot Pe_{wo}} = \frac{x}{R \cdot (Re_D \cdot Pr)_{wo}}, \qquad \theta = \frac{T - T_1}{T_w - T_i} \tag{10}$$

Substituting these variables into the original equations, the nondimensional problem is formulated as follows:

$$\frac{\partial \theta}{\partial x_1} = \frac{\alpha^*}{\left[1 - r_1^2\right]} \cdot \left[\frac{\partial}{\partial r_1}\left(f \cdot \frac{\partial \theta}{\partial r_1}\right) + \frac{f}{r_1} \cdot \frac{\partial \theta}{\partial r_1}\right] \tag{11}$$

with

$$\theta = 0 \quad \text{at} \quad x_1 = 0, \qquad \frac{\partial \theta}{\partial r_1} = 0 \quad \text{at} \quad r_1 = 0$$

$$\theta = 1 \quad \text{at} \quad r_1 = 1$$

where α^*, the normalized thermal diffusivity, is given by the following:

$$\alpha^* = 1, \qquad \theta < ML$$

$$\alpha^* = \alpha_{wp}/\alpha_{wo} = Cp_{wo}/Cp_{wp}, \qquad ML \le \theta \le \theta_2$$

$$\alpha^* = 1, \qquad \theta < \theta_2$$

In the melting range, α^* is given by the following equations for the four different specific heat functions.

Rectangular:

$$\frac{\alpha_{wp}}{\alpha_{wo}} = \frac{Cp_{wo}}{Cp_{wp}} = 1\bigg/\left[1 - Cm \cdot \frac{Cp_s}{Cp_{wo}} + \frac{1}{Ste \cdot Mr}\right] \tag{12}$$

Right triangle:

$$\frac{\alpha_{wp}}{\alpha_{wo}} = 1\bigg/\left\{1 + \frac{2}{Ste \cdot Mr} \cdot \left(\frac{\theta - ML}{Mr}\right) - Cm \cdot \frac{Cp_s}{Cp_{wo}} \cdot \left[1 + \left(\frac{\theta - ML}{Mr}\right) - \left(\frac{\theta_2 - \theta}{Mr}\right)\right]\right\} \tag{13}$$

Left triangle:

$$\frac{\alpha_{wp}}{\alpha_{wo}} = 1\bigg/\left\{1 + \frac{2}{Ste \cdot Mr} \cdot \left(\frac{\theta_2 - \theta}{Mr}\right) - Cm \cdot \frac{Cp_s}{Cp_{wo}} \cdot \left[1 + \left(\frac{\theta_2 - \theta}{Mr}\right) - \left(\frac{\theta - ML}{Mr}\right)\right]\right\} \tag{14}$$

Sinusoidal:

$$\frac{\alpha_{wp}}{\alpha_{wo}} = 1\bigg/\left[1 + \frac{\pi}{2} \cdot \left(\frac{1}{Ste \cdot Mr} - Cm \cdot \frac{Cp_s}{Cp_{wo}}\right) \cdot \sin \pi \cdot \left(\frac{\theta - ML}{Mr}\right)\right] \tag{15}$$

Note that the following condition must be satisfied in all cases to ensure that the problem is related to the heating/melting process, that is, $T_i \le T_1$:

$$Cm \cdot Cp_s/Cp_{wo} \le 1/Ste \cdot Mr$$

Inspection of Eqs. (11–15) indicates that the governing parameters for the present work are the bulk Stefan number $[Ste = Cp_{wo} \cdot (T_w - T_i)/Cm \cdot h_{fs}]$, the melting temperature range $[Mr = (T_2 - T_1)/(T_w - T_i)]$, the degree of subcooling $[ML = (T_1 - T_i)/(T_w - T_i)]$, and the specific heat ratio $(Cm \cdot Cp_s/Cp_{wo})$.

Verification of Model

It is not possible to obtain an analytical solution for the present work due to the complexity of the normalized thermal diffusivity α^* in Eqs. (12–15). Hence, numerical solutions have been obtained. The differential equation has been converted into a finite difference form using central differences in the radial direction and forward differences in the axial direction. An explicit method was used to solve the set of difference equations. All solutions were obtained using 15 significant digits in the calculations.

As a first step, numerical results were obtained for the case with no phase change, that is, the Graetz solution. Next numerical results were obtained for the case with $ML = 0$, $Ste = 1$, and $Mr = 10^{-4}$. They were compared to the results of Charunyakorn et al.[2] for the same value of Stefan number (and $ML = Mr = 0$). The effects of the other parameters in the model of Charunyakorn et al.[2] (c, $Pe_f \cdot r_p/R$, R/r_p, k_p/k_f) are felt indirectly through the Stefan number and the effective thermal conductivity of the slurry. Solutions were obtained with all four specific heat functions, and the maximum difference between them and the results of Charunyakorn et al.[2] was less than 0.3%. No verification was done with respect to experimental data because available data for laminar flows are limited to the constant wall heat flux problem.[3] Furthermore, because the degree of subcooling were not known exactly for these tests, the data can be used only for qualitative comparisons.[3,4] Extensive grid size (both Δx and Δr) independence tests were conducted to confirm the validity of the numerical results. These were done with the basic parameter values of $ML = 0$, $Ste = 1$, $Mr = 10^{-4}$, and $Cm \cdot Cp_s / Cp_{\text{mix}} = 0.0085$, with grid sizes ranging from $\frac{1}{30}$ to $\frac{1}{60}$ in the r direction and 1/75,000–1/180,000 in the x direction. The differences in the solutions were less than 0.2%, and grid sizes of $\Delta x = 1/75{,}000$ and $\Delta r = 1/40$ were used in most of the parametric studies.

Discussion

Numerical results were obtained using the four different specific heat functions to evaluate the influence of the phase change characteristics on the overall heat transfer process. The parameter values for this study were taken as $Mr = 0.3$, $ML = 0.4$, $Ste = 2$, and $Cm \cdot Cp_s / Cp_{wo} = 0.1$. Figure 2 shows that the differences between the solutions using different specific heat functions are very small, of the order of 4% or less. These results show that the exact nature of the phase change process is not critical in modeling the heat transfer process and simple specific heat functions can be used in numerical modeling. It is necessary only that the latent heat of melting and the melting temperature range be known accurately so that the relevant nondimensional parameters can be evaluated.

The reason for the lack of sensitivity to the shape of the specific heat function as described becomes quite clear when one considers previous work done in a related area by Bart and van der Laag.[6] Their study of latent heat energy storage materials showed that the heat absorption/release characteristics of PCMs with different specific heat functions were quantitatively similar to those in materials with an equivalent rectangular-shaped specific heat curve. The present case is a very similar problem, with the phase change effects being smaller due to the limited concentration of the PCMs in the slurry. Thus, the maximum difference of about 4% between the solutions for the four cases is reasonable.

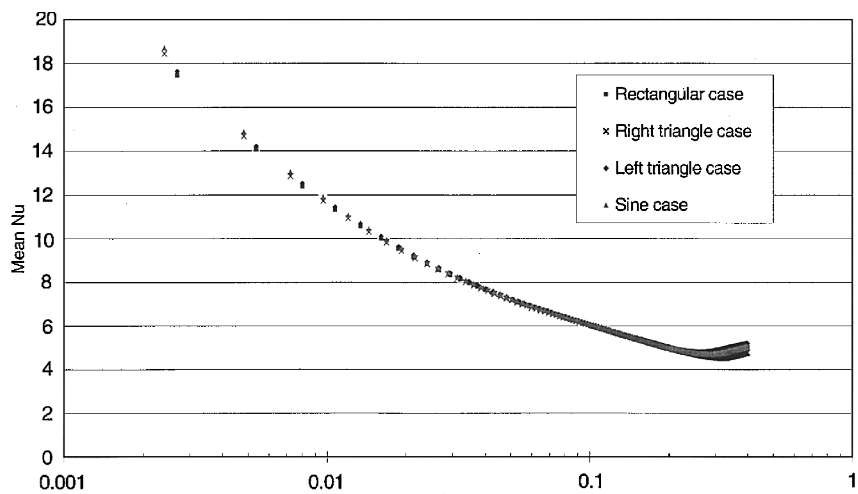

Fig. 2 Comparison of mean Nusselt number variation with four different specific heat functions: $Mr = 0.3$, $ML = 0.4$, $Ste = 2$, and $Cm \cdot Cp_s/Cp_{wo} = 0.1$.

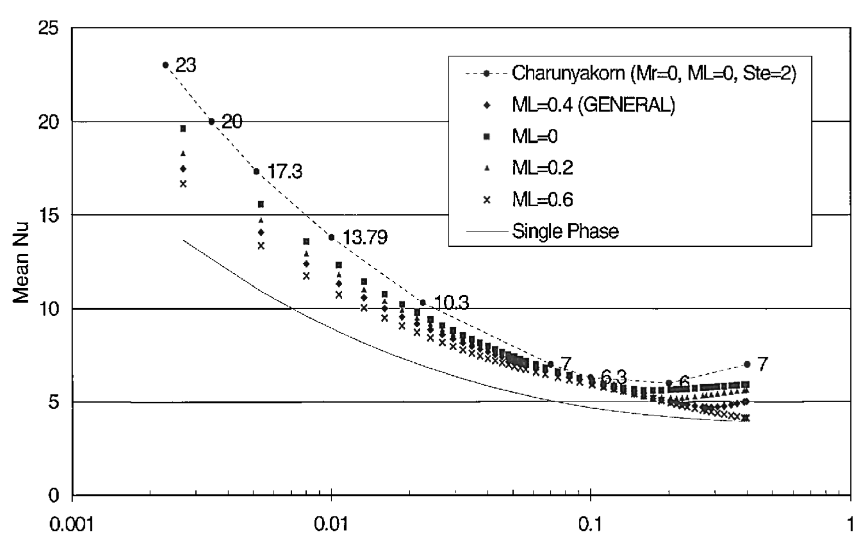

Fig. 3 Mean Nusselt numbers for typical parameter values; variation as a function of the degree of subcooling, ML: $Mr = 0.3$, $Ste = 2$, and $Cm \cdot Cp_s/Cp_{wo} = 0.1$.

Additional work was done to evaluate the effects of the other nondimensional parameters using only the rectangular shape for the specific heat function.[5] Results confirm previous predictions[2,4] that the Stefan number is the most dominant parameter for the present problem. This is true for Stefan numbers up to approximately 10, beyond which the heat transfer enhancement is driven mainly by the effect of the particle-induced microconvection. The degree of subcooling is another important parameter, as reported by Zhang and Faghri,[4] with the heat transfer enhancement increasing as the degree of subcooling decreases. The melting temperature range does not have as strong an impact as the other two parameters, with the heat transfer enhancement remaining approximately constant for Mr values between 0 and 0.3. Thus, small levels of impurities in the PCM should not have a significant effect on the heat transfer process as long the Mr value remains in this range. Microencapsulation of the PCM (resulting in the presence of some level of impurities) and/or the use of industrial-grade chemicals (as opposed to laboratory-grade materials) should, therefore, be acceptable in many practical applications. The final parameter, namely, the specific heat ratio, does not affect the heat transfer characteristics significantly because the specific heat ratio is typically very small compared to the product $1/(Ste \cdot Mr)$ in Eqs. (12–15). As observed by Goel et al.,[3] the overall results also show that the heat transfer enhancements may be substantially lower than those predicted by Charunyakorn et al.[2] when more realistic values of degree of subcooling and melt temperature range are taken into account (Fig. 3).

Conclusion

An effective specific heat model for heat transfer in PCM slurries has been developed, and the problem of laminar heat transfer to a PCM slurry in a circular duct with constant wall temperature has been investigated. The governing parameters are the bulk Stefan number, the degree of subcooling, the dimensionless melting temperature range, and the specific heat ratio. Results show that the exact form of the specific heat function is not critical as long as the latent heat is incorporated correctly within the melting temperature range. The dominant parameters were found to be the bulk Stefan number, the degree of subcooling, and the dimensionless melting temperature range, and the effect of the specific heat ratio is very small.

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